Provable equivalence

Let φ and ψ be formulas of propositional logic. We say that φ and ψ are provably equivalent iff (we write 'iff' for 'if, and only if' in the sequel) the sequents φ ψ and ψ φ are valid; that is, there is a proof of ψ from φ and another one going the other way around. As seen earlier, we denote that φ and ψ are provably equivalent by φ ψ.

Note that, by Remark 1.12, we could just as well have defined φ ψ to mean that the sequent $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$ is valid; it defines the same concept.

Examples of provably equivalent formulas are

$\neg(p \wedge q)$ $\neg q \vee$ $\neg p$ $\neg(p \vee q)$ $\neg q \wedge \neg p$

$p \rightarrow q$ $\neg q \rightarrow \neg p$ $p \rightarrow q$ $\neg p \vee q$

$p \wedge q \rightarrow p$ $r \vee \neg r$ $p \wedge q \rightarrow r$ $p \rightarrow (q \rightarrow r)$.

The reader should prove all of these six equivalences in natural deduction.